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Advances in Water Resources 30 (2007) 1873–1882

Advances in Water Resources

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Review of Gould–Dincer reservoir storage–yield–reliability estimates

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Received 9 November 2006; received in revised form 8 February 2007; accepted 8 February 2007
Available online 21 February 2007

Abstract

The Gould–Dincer suite of techniques (normal, log-normal and Gamma), which is used to estimate the reservoir capacity–yield–reliability (*S–Y–R*) relationship, is the only known available procedure in the form of a simple formula, based on annual streamflow statistics, that allows one to compute the *S–Y–R* relationship for a single storage capacity across the range of annual streamflow characteristics observed globally. Several other techniques are available but they are inadequate because of the restricted range of flows on which they were developed or because they are based on the Sequent Peak Algorithm or are not suitable to compute steady-state reliability values. This paper examines the theoretical basis of the Gould–Dincer approach and applies the three models to annual streamflow data for 729 rivers distributed world-wide. The reservoir capacities estimated by the models are compared with equivalent estimates based on the Extended Deficit Analysis, Behaviour analysis and the Sequent Peak Algorithm. The results suggest that, in the context of preliminary water resources planning, the Gould–Dincer Gamma model provides reliable estimates of the mean first passage time from a full to empty condition for single reservoirs. Furthermore, the storage estimates are equivalent to deficits computed using the Extended Deficit Analysis for values of drift between 0.4 and 1.0 and the values are consistent with those computed using a Behaviour simulation or a Sequent Peak Algorithm. Finally, a sensitivity analysis of the effect on storage of the four main streamflow statistics confirms that the influential ones are mean and standard deviation, while effects of skew and serial correlation are orders of magnitude lower. This finding suggests that the simple reduced form of the Gould–Dincer equation may profitably be used for regional studies of reservoir reliability subject to climate change scenarios based on regional statistics, without having to perform calculations based on time series, which may not be easily obtained.

Keywords: Reservoir theory; Storage–yield analysis; Dincer; Gould–Dincer

1. Introduction

During the preliminary design phase of a water resources development or for a reconnaissance review of the yield from one or more reservoirs, it is useful to have available a technique that one can use to examine the reservoir capacity, yield and reliability (*S–Y–R*) relationship for a single reservoir. Sometimes, particularly in countries with poor infrastructure, the historical flow record is short or even unavailable yet a preliminary *S–Y–R* relationship is required. Furthermore, for water resources assessments and to assess the impact of climate change on hydrology at a continental scale, a simple procedure using annual statistics is required in which change in precipitation and temperature can be related to reservoir characteristics.

There are very few general *S–Y–R* procedures that are available for such preliminary analyses. Some are based on limited empirical information and, therefore, are inadequate for wide hydrologic application. Computer-based methods like Behaviour analysis are unsuitable where flow

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doi:10.1016/j.advwatres.2007.02.004

records are short or not available. For reconnaissance analysis in countries with inadequate hydrologic records, reservoir capacities can only be sized on the basis of an estimate of the mean flow and its variability. The authors are aware of only one technique – Gould–Dincer (G–D) suite of equations – that potentially can be applied using annual streamflows across the whole spectrum of global hydrology. It should be noted here that there are preliminary techniques that can be used for a restricted range of hydrology (Buchberger and Maidment [1]) or are able to estimate the reservoir capacity for no-failure yield (Vogel and Stedinger [24]; and Phien [17]) but cannot estimate capacity–steady state reliability relationships. The authors believe the G–D approach is the only suitable candidate to examine such relationships and is, therefore, the subject of this paper.

Although the G–D approach is widely recommended for use as a preliminary *S–Y–R* technique (for example Gould [4], Teoh and McMahon [19], McMahon and Adeloye [7]), the techniques have not been fully reviewed. The first author examined one aspect of the approach, but the analysis was based on data limited to Australia and Malaysia (McMahon and Mein [8]; Teoh and McMahon [19]). With the much larger data set of annual streamflows considered here – 729 rivers globally with at least 25 years of continuous clean data – we believe it is opportune to carry out a more detailed review of the G–D techniques. This paper reviews the Gould–Dincer suite of equations that can be used to establish the relationship between reservoir capacity, yield and reliability over the most extensive range of streamflow records, and the widest range of climates, that has ever been considered.

Following this introduction, we describe briefly in Section 2 the global annual streamflow data set used to assess the *S–Y–R* techniques. Next, we outline the background theory of the three variant forms (normal, lognormal and Gamma) of the Gould–Dincer approach. In Section 4, we compare the mean first passage time from a full to empty reservoir (m_{0c}) as defined theoretically by Pegram [14] for a finite reservoir with an approximate equivalent recurrence interval calculated using the G–D normal model. In the next section, the G–D approach is applied to the data set and is compared to three alternate *S–Y–R* techniques to assess the performance of the G–D methods. Parameter sensitivity in estimates of reservoir capacity based on the Gould–Dincer model is explored in Section 6. There it is found that the effects of skew and serial correlation are orders of magnitude lower than those of changes of the mean and standard deviation. This finding suggests that the simple reduced form of the Gould–Dincer equation may profitably be used for regional water resources assessments and in studies of reservoir reliability subject to climate change scenarios based on regional statistics, without having to perform calculations based on time series, which may not be easily obtained. Relevant conclusions are drawn in Section 7.

2. Streamflow data

The streamflow data set used in this analysis consists of historical streamflows for 729 rivers with 25 or more years of continuous annual data. The rivers are not regulated by reservoirs nor are they affected by diversions upstream of the gauging stations. The location of the 729 rivers suggests that the data are reasonably well distributed world-wide. Details of the data can be found in [10,12,13].

3. Gould–Dincer approach

The Gould–Dincer approach, which was offered to the first author by C.H. Hardison in 1966, is a modification of a method for reservoir storage–yield analysis derived by Professor T. Dincer, Middle East Technical University, Turkey. The Dincer method assumed the reservoir inflows were normally distributed and serially uncorrelated. In 1964, Gould [4] independently derived a similar reservoir storage–yield relationship but incorporated inflows that were Gamma distributed. This method has become known as the Gould Gamma method (McMahon and Adeloye [7]). To apply this method to skewed flows Gould [4] provided a manual adjustment to modify normal flows to Gamma distributed flows. Vogel and McMahon [23] proposed that the Wilson–Hilferty transformation [27] be used instead of the cumbersome Gould procedure to deal with skewed flows and derived an adjustment to storage as a result of auto-correlation which produced a result identical to that of Phatarfod [16], but used a completely independent approach. A further variation of the Gould–Dincer approach that allows for lognormal inflows was offered to the first author by G. Annanadale in 2004. To distinguish among these three variations of the Gould–Dincer approach we have labelled them: Gould–Dincer Normal (G–DN), Gould–Dincer Gamma (G–DG) and Gould–Dincer Lognormal (G–DLN). The following summarizes the methodology.

3.1. Gould–Dincer Normal (G–DN)

The equation representing G–DN model is developed as follows. Assuming normally distributed and independent annual flows (mean μ and standard deviation σ), consecutive n-year inflows (i.e., the sum of n consecutive annual flows) into a reservoir can be defined as:

$$n\text{-year mean, } \mu_n = n\mu \tag{1}$$

$$n\text{-year standard deviation, } \sigma_n = \left(\sum^{n} \sigma^2\right)^{0.5} = \sigma\sqrt{n} \tag{2}$$

During a critical period (i.e., a period during which the reservoir contents decline from full to empty) of length n:

$$S_{n,p} = D_n - X_{n,p} \tag{3}$$

where $S_{n,p}$ is the depletion (thought of as a positive quantity) of an initially full reservoir at the end of n years without having spilled, $D_n = \alpha n\mu$ is the target draft over n years, $X_{n,p}$ is the n-year inflow with a probability of non-exceedance of 100p% and α is a constant draft defined as a ratio of μ. Assuming inflows are normally distributed

$$X_{n,p} = n\mu + z_p\sigma_n \tag{4}$$

where z_p is the standardised normal variate at 100p% probability of non-exceedance ($z_p < 0$ because we are looking at inflows below the mean). To obtain the maximum storage required to supply the draft, combine Eqs. (3) and (4) and differentiate with respect to n; this gives the required capacity C to meet the target draft for 100p% probability of non-exceedance, i.e., for $100(1-p)$% reliability, and the equivalent critical period n_{crit} in years as follows:

$$n_{\text{crit}} = \frac{z_p^2}{4(1-\alpha)^2}\text{Cv}^2 \tag{5}$$

which, after back-substitution into Eq. (3), gives:

$$C = \frac{z_p^2}{4(1-\alpha)}\text{Cv}^2\mu \tag{6}$$

where Cv is the coefficient of variation of annual inflows to the reservoir. n_{crit} is the period for the reservoir of capacity C to empty from an initially full condition.

By substituting $\frac{1-\alpha}{\text{Cv}} = m$ in Eq. (6), we obtain the dimensionless relationship:

$$K = \frac{z_p^2}{4m} \tag{7}$$

where K is the standardised storage (reservoir capacity divided by the standard deviation of annual flows) and m is known as drift [14,21] or standardized net inflow [5], in other words, m is the inverse of the coefficient of variation of net inflow. Note that for a given reliability, K reduces as m increases.

To account for the auto-correlation effect on reservoir capacity one can adjust the reservoir capacity computed from Eq. (6) by $(1+\rho)/(1-\rho)$ as follows [16,23]:

$$C = \frac{z_p^2}{4(1-\alpha)}\text{Cv}^2\mu\frac{1+\rho}{1-\rho} \tag{8}$$

where ρ is the lag-one serial correlation coefficient.

It is noted that probability p is the probability that inflows into the reservoir will be just sufficient to allow the reservoir to meet the targeted draft with reliability $(1-p)$. In terms of Pegram's definitions of failure [14] discussed in Section 4, $1/p$ is assumed to be a measure of the mean first passage time from a full to an empty reservoir.

3.2. Gould–Dincer Gamma (G–DG)

If the inflows are assumed to be Gamma distributed, z_p in Eq. (8) is replaced by:

$$g_p = \frac{2}{\gamma}\left[\left\{1 + \frac{\gamma}{6}\left(z_p - \frac{\gamma}{6}\right)\right\}^3 - 1\right] \tag{9}$$

where g_p is an approximate Gamma variate based on the Wilson–Hilferty transformation [27] (see also Chowdhury and Stedinger [3] for developments relating to the transformation) and γ is the coefficient of skewness of the annual inflows. If the flows are also auto-correlated, then γ needs to be replaced by γ' (Eq. (10)) in Eq. (9). Eq. (10) was first proposed by Thomas and Burden [20] and published by Loucks et al. [6, p. 284]. This correction adjusts γ and is separate from the correction in Eq. (8) which deals with the effect of auto-correlation on reservoir inflows and is independent of the inflow distribution:

$$\gamma' = \gamma\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right) \tag{10}$$

It should be pointed out that the Gamma transformation (Eq. (9)) based on the Wilson and Hilferty transformation breaks down for values of $\gamma > 4$ [9]. This is equivalent to Cv = 1 in the lognormal domain, however, very few values (only five out of the estimated values of γ from the world data-set) are >4.

3.3. Gould–Dincer Lognormal (G–DLN)

If the annual flows are considered to be lognormal, z_p in Eq. (8) can be replaced by Eq. (11) which is a rearrangement of Chow [2, Eq. (8-I-53)]

$$z'_p = \frac{1}{\text{Cv}}\left[e^{z_p\sqrt{\ln(1+\text{Cv}^2)} - 0.5\ln\left(1+\text{Cv}^2\right)} - 1\right] \tag{11}$$

3.4. Attributes, limitations

A major advantage of the Gould–Dincer approach is that it is based on a straight-forward and logical water balance of simultaneous inputs and outputs of a storage reservoir. Computationally, it is simple and, although the basic formulation assumes inflows are independent, the storage estimates can be adjusted to take the auto-correlation into account as provided in Eq. (8). It has been shown elsewhere [19] that for carry-over or over-year storages the Gould–Dincer approach provides satisfactory estimates of reservoir capacity. This issue is examined further in Sections 4 and 5.

A limitation of G–D models relates to the definition of probability of failure (emptiness) (Pf). From Eq. (3), the probability of failure is defined as the failure of the n-year inflow to occur with a probability of non-exceedance of 100p%. Given that the unit of time in a G–D analysis is a year, $1/p$ can be likened to and, for our analysis, has been assumed equivalent to the mean first passage time from a full reservoir to an empty condition. The theory does not allow for failures beyond the first failure. The complement of Pf is an approximate measure of the reliability to meet the target draft from a full condition.

The procedure is restricted to annual inflows and, hence, carryover storage. As a general rule reservoirs with $m < 1$ operate as over-year or carry-over storages and as within-year systems if $m \geqslant 1$ [22,25]. Vogel et al. [26] suggested that $m < \mathrm{Cv}$ and $m \geqslant \mathrm{Cv}$ maybe a more appropriate guide. However, Montaseri and Adeloye [11] argue that other variables in addition to m and Cv, such as reliability, may be needed to classify reservoirs as carry-over or within-year storages. In this paper, we have adopted the simple rule that $m < 1$ or $m \geqslant 1$ and a further check was carried out by ensuring that n_{crit} (Eq. (5)) was greater than 1 year.

Similar to some other S–Y procedures, net evaporation losses cannot be explicitly taken into account, but procedures are available [7] to adjust the storage size once the capacity has been determined using only inflows and drafts.

4. The adequacy of the Gould–Dincer approach

A specific reservoir size computed using the Gould–Dincer approach is based on the definition of probability of non-exceedance of inflows (Eq. (3)). As noted earlier this has been assumed equivalent to the mean first passage time from a full to an empty reservoir (m_{0c}) as defined by Pegram [14]. Employing numerical quadrature to solve the integral storage equations and using the deferred approach to the limit, Pegram [14] obtained a range of results in reservoir storage reliability with known error bounds. Choosing finite reservoirs with a range of standardized capacities, he selected inputs described by Normal and lognormal distributions with a range of values of drift, coefficient of skewness and lag-one serial correlation. Of interest here are the results for reservoirs fed with independent inputs, obtaining their reliability as measured by m_{00}, the mean recurrence time of emptiness and m_{0c}, the mean first passage time from full to empty; these were all computed to a precision of at least three significant figures.

In Fig. 1, the mean first passage time to empty from full, m_{0c}, is plotted against the mean recurrence time of emptiness, m_{00}, i.e. the mean time between failures, based on results from Pegram [14]. The figure is based on the standardised storage and drift with values ranging mainly from 0.5 to 8 (for K) and 0–1 (for m). For practical purposes and for annual flow data, values up to perhaps 1000 time units are relevant. It can be seen that as drift, m, increases m_{0c} approaches m_{00}. This is reasonable given that as drift increases a reservoir tends to spill more frequently, experiencing rarer empty events, so that the first passage time to empty from full becomes indistinguishable from the times between occurrence of emptiness; it is noted in the figure that the means of the two types of passage time are very close for drifts above about 0.4.

To test the adequacy of the Gould–Dincer Normal procedure (Eq. (7)), values of the mean first passage time from a full to empty condition were computed using Eq. (7) for appropriate $1/p$ values and were compared with Pegram's exact solution values of m_{0c} [14] for the equivalent drift (m) and standardized storage values (K). The comparison is plotted in Fig. 2 for values of time units less than 1000. In the figure, m_{0c} values for $m = 0.3$, 0.5, 0.7 and 0.9 were interpolated from Table III in Pegram [14]. The other values were read directly from the table. The slope of the regression without intercept is 1.00035. From this analysis it can be concluded that G–DN for $m < 1$ is a satisfactory estimate of the mean first passage time from a full reservoir to an empty condition. We are unable, however, to independently test the adequacy of the other two forms of the G–D approach namely G–D Gamma and G–D Lognormal, as the mean first passage times based on Gamma and Lognormal inflows are not available in Pegram [14]. The adequacy of these two latter variations along with G–DN are examined in the following section.

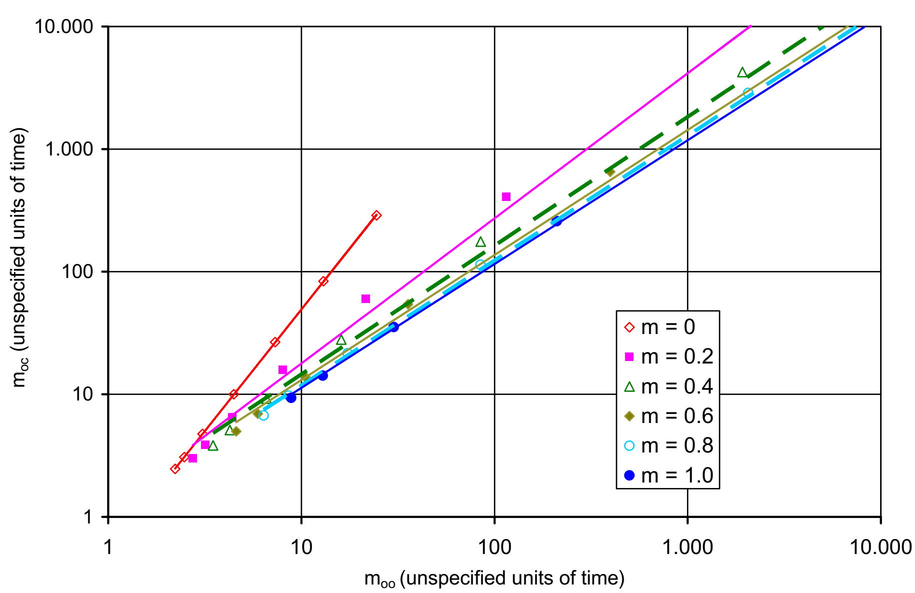

Fig. 1. Plot of m_{0c} (mean first passage time to empty from full storage) compared with m_{00} (mean recurrence time of emptiness) based on Pegram's [14] results for finite reservoirs fed by normally distributed inflows.

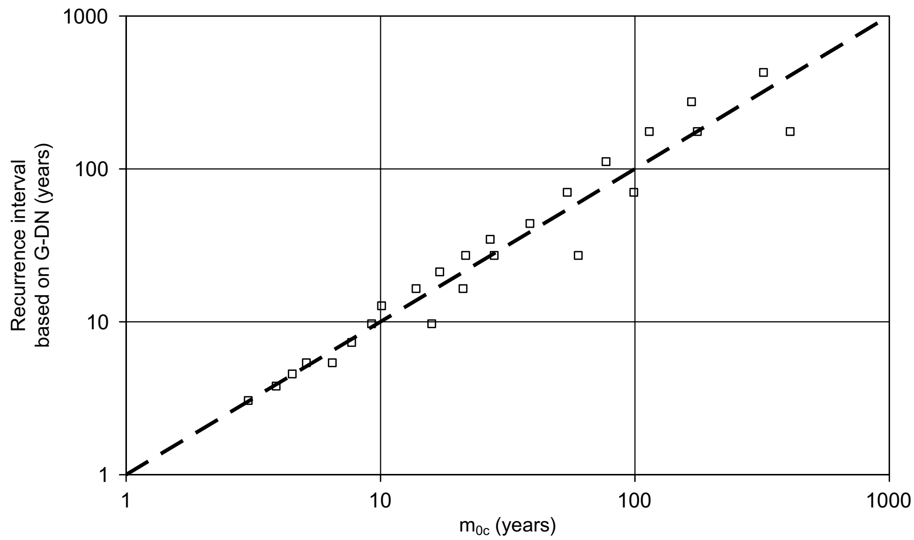

Fig. 2. Comparison of m_{0c}, mean first passage time from full reservoir to empty, for Gould–Dincer Normal procedure compared with Pegram's exact solution [14] using a range of drifts ($0.2 \leqslant m \leqslant 0.9$), standardised capacities in the range $0.25 \leqslant C \leqslant 8$ and $n_{crit} > 1$. The 1:1 line is dashed.

5. Application of Gould–Dincer to world data

In this section, we compare the reservoir capacities estimated by the Gould–Dincer approach with those estimated by the Extended Deficit Analysis (EDA), Behaviour analysis and the Sequent Peak Algorithm (SPA). The EDA technique estimates for a river the deficit of providing a given draft (from an initially full semi-infinite reservoir) at a given level of reliability. It is based on estimating from the historical flows a series of independent deficits that are assessed in a manner similar to a partial flood frequency analysis. Details of the technique are set out in Pegram [15] (also see [7] for an explanation). For Behaviour analysis, failure is measured as the percentage of time (annual time units in this study) that the reservoir is empty. It is a technique that tracks the volumetric content of a finite reservoir by carrying out a water balance of inputs (inflows) and outputs (yields or reservoir releases and spills and other losses including net evaporation). In the analysis in this paper, losses are not taken into account. In contrast to the other techniques, SPA estimates the firm yield which is the yield that can be met over a particular planning period with a specified no-failure reliability. Based on historical streamflows it is an automated version of the Rippl mass curve procedure [20] and has been used to estimate reservoir capacities worldwide. In a complementary (as yet unpublished) paper based on the same 729 annual river flows adopted in this paper, we show that these three techniques produce consistent estimates of reservoir capacity, with SPA and EDA procedures computing virtually the same capacities.

5.1. Gould–Dincer Gamma versus EDA

In order to apply the Gould–Dincer approach, we have assumed in this application that the global annual flows are Gamma distributed. Our analysis to be published elsewhere would suggest this is not an unreasonable assumption.

Although the G–D approach and EDA are based on different probabilistic statements (G–D estimates are assumed to approximate the mean first passage time to empty from a full storage and EDA estimates the mean recurrence time of emptiness), it is instructive to compare the storage estimates generated by both approaches. We are able to do this for the range of drift $0.4 < m < 1$ where the mean first passage time approximates the mean recurrence time of emptiness (see Fig. 1 and Section 4). For this range of drift we compared for the global rivers the G–D Gamma storage estimates using Eqs. (8)–(10) with the EDA values, both based on $\alpha = 0.75$ (that is, a draft of 75% of the mean historical flow) and restricting the applications to the 305 rivers with $\gamma < 4.0$ (see Section 3). In addition, we ensured that n_{crit} computed from Eq. (5) > 1 year. The G–DG estimates are for 99% annual time reliability and EDA values are for a recurrence interval of 100 years.

In order to assess the similarity of the capacity estimates between the two procedures, a weighted least squares (WLS) regression was applied, with weights proportional to record lengths, and excluding the outlier indicated in Fig. 3 with a cross. The overestimate by G–DG relative to the EDA value for this (outlier) river record is mainly due to an annual auto-correlation value of 0.74. This value is in the upper 1% of ρ values in the data set and the storage estimate is 6.7 times larger (based on the adjustment given in Eq. (8)) compared with a storage estimate for $\rho = 0$. The slope of the WLS regression is 0.99. From this analysis we can conclude that G–DG and EDA within the range adopted in Fig. 3 provide similar estimates of reservoir capacity for all practical purposes.

5.2. Gould–Dincer Gamma versus Behaviour analysis

Fig. 4 explores the relationship between Gould–Dincer Gamma and a Behaviour analysis for $\alpha = 0.75$ and restricting the applications to rivers with $\gamma < 4.0$, $m < 1.0$ and

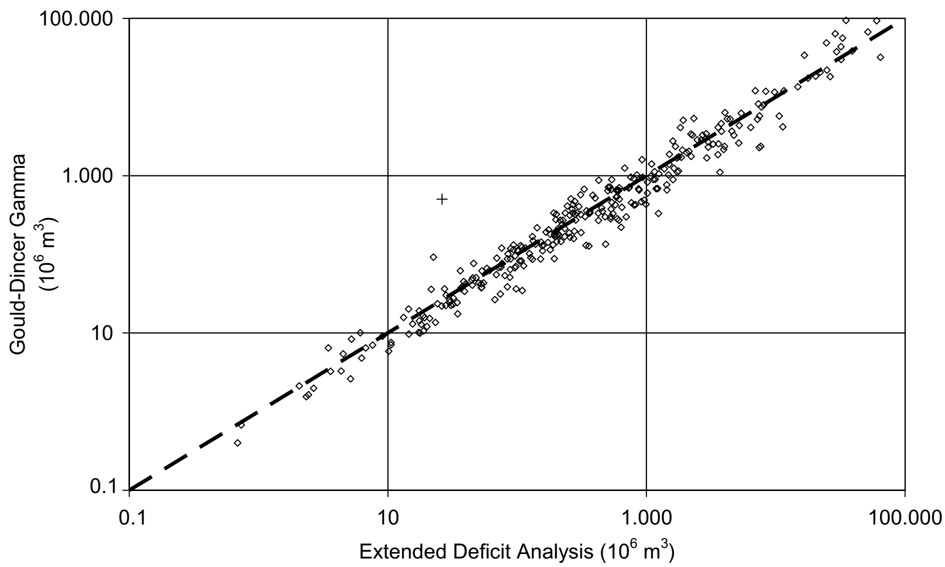

Fig. 3. Comparison of Gould–Dincer Gamma procedure for $\alpha = 0.75$, 99% annual time reliability with storage estimates using the Extended Deficit Analysis for $\alpha = 0.75$ and 100 years average recurrence interval. The analysis was restricted to $0.4 < m < 1.0$, $n_{crit} > 1$ year and annual coefficient of skewness of streamflow <4.0. The 1:1 line is dashed.

$n_{crit} > 1$ year. The analysis is for annual data and for 95% annual reliability. (We adopted 95% reliability to ensure that with 25 or more years of data, at least one failure per simulation would be recorded during a Behaviour simulation.)

The figure shows a strong relationship between the two estimates. The slope of the WLS regression, without an intercept term, is 0.958 which is statistically different from unity at the 5% level of significance. A slope less than one is expected because the definitions of failure adopted in the two procedures are different and result in the Behaviour analysis estimates of reservoir capacity being larger than the G–DG estimates. In this paper the adopted failure criterion for the Behaviour analysis is the one used in water engineering practice. It is the proportion of time units during simulation the reservoir fails to meet the target demand, whereas for G–DG the criterion is an approximate estimate of the mean first passage time from full to empty which approximates the mean recurrence time of emptiness for $0.4 < m < 1.0$ (Section 4). Considering for the moment the mean recurrence time of emptiness, it therefore follows that for each G–DG failure (emptiness), there will be a corresponding period of failure observed during the equivalent Behaviour analysis. However, for

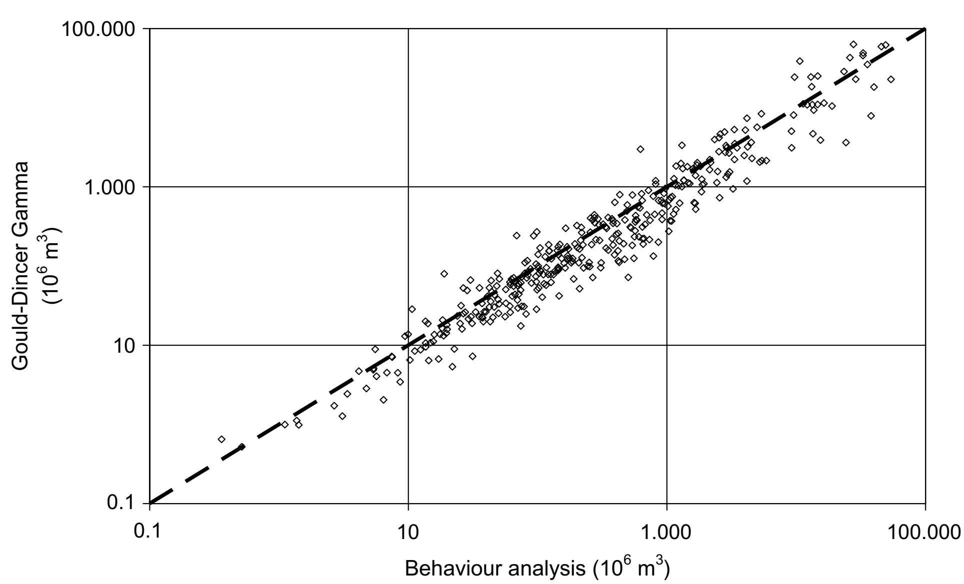

Fig. 4. Comparison of Gould–Dincer Gamma reservoir estimates with Behaviour estimates for $\alpha = 0.75$ and 95% reliability. The analysis was restricted to $0.4 < m < 1.0$, $n_{crit} > 1$ year and annual coefficient of skewness of streamflow <4.0. The 1:1 line is dashed.

the latter each failure may last longer than one time unit. As a result, for the same reliability criterion, say 95% (or 5% failure), there will be more time units of failure associated with the Behaviour simulation than for G–DG. This means the reservoir capacity estimated using G–DG will be smaller than the Behaviour capacity for the same reliability or failure condition. For reservoir capacities where $m < 0.4$, we observe from Fig. 1 that as $m_{0c} > m_{00}$ this effect will be amplified.

Using the world data set the analysis confirms that the G–DG procedure provides reservoir capacity estimates which for the same numerical value of probability of failure are smaller (on average about 25%) than capacities computed using a Behaviour analysis. This result is consistent with the definitions of failure adopted for the two procedures.

5.3. Gould–Dincer Gamma versus SPA

Figs. 2 and 3 confirmed that Gould–Dincer Gamma is able to reliably estimate the reservoir capacity for given target draft and probability of failure (emptiness) defined as the mean first passage time from a full to empty storage. Fig. 4 further showed that the G–DG reservoir capacity estimates were consistent with those estimated using a Behaviour analysis. The purpose of Fig. 5 is to observe, firstly, the difference in storage estimates using the three distributions associated with the Gould–Dincer suite of equations namely G–DN, G–DG and G–DLN and, secondly, the relationship between estimates of reservoir capacity by the G–D procedures and SPA.

The data plotted in Fig. 5 are for the G–DN, G–DG and G–DLN and the SPA estimates based on $\alpha = 0.75$, for $m < 1$ and $n_{crit} > 1$ year. In order to provide an appropriate comparison between the G–D and SPA estimates, the probability of failure in the G–D analysis was defined as $1/N$ where N is the number of years in the historical record.

Several observations follow from the figure. Firstly, the regression slope coefficients being 1.021, 1.006 and 1.013 for G–DN, G–DG and G–DLN respectively are not significantly different from unity at the 5% level. Secondly, the G–D estimates as a whole contain the 1:1 line with the G–DN, G–DG and G–DLN values, on average across the range of capacities, being respectively 34% larger, 32% smaller and 47% smaller than the equivalent SPA value. (The intercepts cause the averages to vary from each other; the trend-lines through the respective sets have been omitted as they are swamped by the data.) This result emphasizes the importance of choosing the correct probability distribution function for the reservoir inflows when one is computing storage estimates using the Gould–Dincer suite of equations.

6. Parameter sensitivity in estimates of reservoir capacity estimation

In this section, we explore the impact of parameter sensitivity in estimating reservoir capacity by considering the effect of an equivalent change in the four parameters (μ, σ, γ and ρ) on reservoir sizing. To do this we use the Gould–Dincer Gamma procedure and determine theoretically the effect of making a small change separately to each of the four parameters in Eqs. (8)–(10). The resulting marginal (akin to partial differential) error relationships are given as follows:

$$\frac{\Delta C}{C} = \frac{1}{\alpha - 1}\frac{\Delta\mu}{\mu} \tag{12}$$

$$\frac{\Delta C}{C} = 2\frac{\Delta\sigma}{\sigma} \tag{13}$$

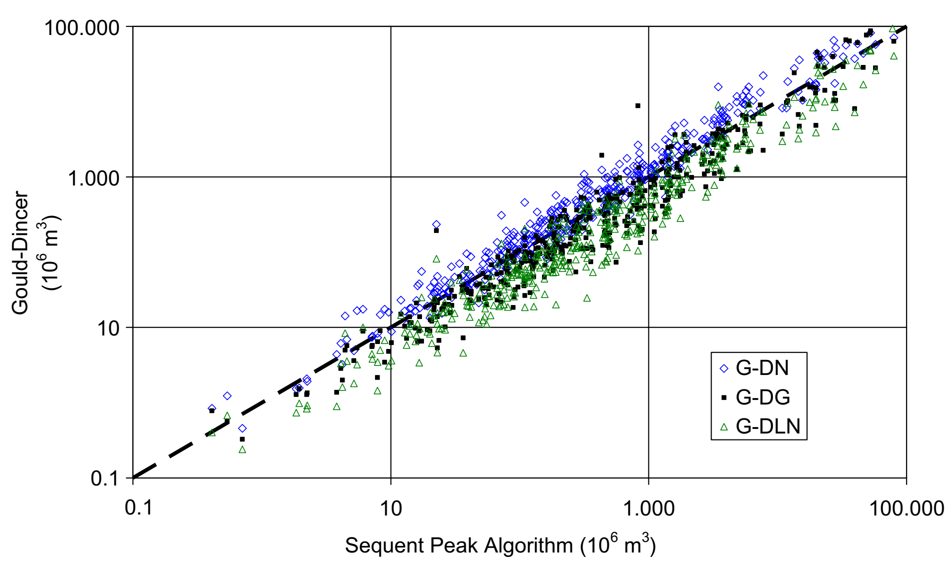

Fig. 5. Comparison of reservoir capacities for $\alpha = 75\%$ target draft based on the three Gould–Dincer alternatives compared with Sequent Peak Algorithm. The analysis was restricted to $m < 1.0$ and $n_{crit} > 1$ year. Probability of failure for G–D estimates based on $1/N$. The 1:1 line is dashed.

$$\frac{\Delta C}{C} = \left(\gamma\left(zp - \frac{\gamma p^2}{3}\right)\frac{\left\{1 + z\frac{\gamma p}{6} - \frac{\gamma^2 p^2}{36}\right\}^2}{\left[\left\{1 + z\frac{\gamma p}{6} - \frac{\gamma^2 p^2}{36}\right\}^3 - 1\right]} - 2\right)\frac{\Delta\gamma}{\gamma} \tag{14}$$

$$\frac{\Delta C}{C} = \rho\Bigg[\Bigg(\left\{1 + \frac{gz}{6} - \frac{g^2}{36}\right\}^2\left(z - \frac{g}{3}\right) \times \left[\left\{1 + \frac{gz}{6} - \frac{g^2}{36}\right\}^3 - 1\right]^{-1} - \frac{2}{g}\Bigg) \times 3\gamma\rho(1-\rho)(1-\rho^2)^{-2.5} + \frac{2}{(1-\rho^2)}\Bigg]\frac{\Delta\rho}{\rho} \tag{15}$$

where Δ represents a small change in the variable, z is the standardised normal variate and g and p (both functions of serial correlation) are defined in Appendix A. The derivation of these relationships is outlined in Appendix A.

Example results are shown in Table 1 for a +10% change in μ, σ, γ and ρ, and adopting median values from the global data set of $\gamma = 0.55$ and $\rho = 0.1$. We carried out the analysis for a time reliability value of 95% ($z = -1.645$) and a target draft ratio $\alpha = 75\%$ (i.e. 75% of the historical mean). Thus for these typical values, a +10% increase in each parameter, considered individually, results in −40.0%, +20.0%, −2.52% and +1.95% change in estimated storage capacity respectively. For a value of $\gamma = 1.0$ (29% of values in the global data set are greater than this value), the effect on storage of a 10% increase in γ is a reduction in storage estimate of 5.50%. We also note from Eqs. (12)–(15) that changing the draft ratio α only produces changes associated with the mean and not the other parameters.

These results are of particular interest to those wishing to investigate the impact of climate change on streamflow and the consequential effect on reservoir yield. This theoretical approach is in contrast to using empirical models like those adopted by Vogel et al. [25] in the northeastern United States. As our understanding of the impact of climate change on hydrology at the annual scale improves, through techniques like precipitation elasticity of streamflow [18], information from exercises similar to the above will inform water managers to the likely impact of climate change on reservoir yield. Furthermore, as our understanding of how climate change affects rainfall variability and hence streamflow variability, such effects on reservoir yield can be explored.

Table 1
Change in storage estimate as a result of +10% change in Gould–Dincer Gamma parameters for reliability of 95% ($z = -1.645$), $\alpha = 75\%$, $\gamma = 0.55$ and $\rho_1 = 0.1$

Parameter	+10% change in parameter
μ	−40.0%
σ	+20.0%
γ	−2.52%
ρ	+1.95%

7. Conclusions

Arising out of these analyses a number of conclusions follow.

1. The Gould–Dincer formulas which are based on a simple water balance of storage content plus inflow less draft can accommodate normal, Gamma and lognormal inflows.
2. Theoretically as drift m gets larger and approaches unity (effectively when $m > 0.4$), the mean first passage time from a full to an empty reservoir approaches the mean recurrence time of emptiness for finite reservoirs fed by independent normal inflows.
3. Mean first passage times from a full to an empty reservoir calculated using the Gould–Dincer normal procedure were not significantly different to those calculated by the Pegram [14] exact solution, for a range of drifts and standardized storage capacities.
4. For conditions in which the Gould–Dincer procedure is applicable, reservoir capacities estimated by Gould–Dincer Gamma are, for all practical purposes, similar to those using Extended Deficit Analysis.
5. Gould–Dincer Gamma estimates were also compared with capacity estimates based on a Behaviour analysis. The analysis showed that for the same numerical value of probability of failure the G–DG procedure provides reservoir capacity estimates that, on average, are about 25% smaller than capacities computed using a Behaviour analysis. This observation is consistent with the definitions of failure adopted for the two procedures.
6. Reservoir capacities determined by the Gould–Dincer formulae were, on average, 34% larger and 32% and 47% smaller for the G–DN, G–DG and G–DLN models respectively compared with capacities obtained from the SPA technique, in which the reciprocal of the length of record was used as the probability of failure.
7. Using the Gould–Dincer Gamma model to represent the storage–yield relationship, a sensitivity analysis for a typical set of values taken from the global data set and based on a 10% change in their value was carried out. For a draft ratio $\alpha = 0.75$, a +10% error in the mean or standard deviation of flows resulted in storage estimates being underestimated by 40% or overestimated by about 20% respectively. However, much smaller relative changes in storage resulted for a 10% change introduced into the coefficient of skewness (−2.5%) or the auto-correlation (2.0%). Because these are an order of magnitude lower than the effects of mean and standard deviation, they can be considered as second order effects. This result suggests that the reduced forms of the G–D equations (Eqs. (6) and (7)) might profitably be used for assessing the effect of climate change scenarios on regional storage reliability.

Acknowledgements

We thank the Department of Civil and Environmental Engineering, the University of Melbourne and the Australian Research Council Grant DP0449685 for financially supporting this research. Our original streamflow data set was enhanced by additional data from the Global Runoff Data Centre (GRDC) in Koblenz, Germany. Streamflow data for Taiwan and New Zealand were also provided by Dr. Tom Piechota of the University of Nevada, Las Vegas. Professor Ernesto Brown of the Universidad de Chile, Santiago kindly made available Chilean streamflows. Thanks for South African reservoir data are also due to the Department of Water Affairs and Forestry with help from consultants WRP who extracted the details.

We are also grateful to Dr. Senlin Zhou of the Murray-Darling Basin Commission who completed early drafts of the computer programs used in the analysis.

Appendix A. Sensitivity of the Gould–Dincer Gamma reservoir storage–yield relationships to inflow statistics

The Gould–Dincer Gamma storage equation (combining Eqs. (8)–(10)) is

$$C=\frac{\sigma^2}{(\mu-D)\gamma^2}\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right)^{-2}\times\left[\left\{1+\frac{\gamma z}{6}\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right)-\frac{\gamma^2}{36}\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right)^2\right\}^3-1\right]^2\left(\frac{1+\rho}{1-\rho}\right) \quad \text{(A1)}$$

where α in Eq. (2) is replaced by D/μ where D is the target draft.

To examine the sensitivity of a storage estimate to the *mean* μ, we consider Eq. (A1) and let

$$k=\frac{\sigma^2}{\gamma^2}\left[\left\{1+\frac{\gamma z}{6}\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right)-\frac{\gamma^2}{36}\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right)^2\right\}^3-1\right]^2\times\left(\frac{1+\rho}{1-\rho}\right) \quad \text{(A2)}$$

$$C=k(\mu-D)^{-1} \quad \text{(A3)}$$

$$\frac{\partial C}{\partial\mu}=-k(\mu-D)^{-2}\rightarrow\frac{\partial C}{C}=\frac{\mu}{D-\mu}\frac{\partial\mu}{\mu} \quad \text{(A4)\&(A5)}$$

$$\frac{\partial C}{C}=\frac{1}{\alpha-1}\frac{\partial\mu}{\mu} \quad \text{(A6)}$$

To examine the sensitivity of a storage estimate to the *standard deviation* σ, we consider Eq. (A1)

$$C=\frac{\sigma^2}{(\mu-D)\gamma'^2}\left[\left\{1+\frac{\gamma' z}{6}-\frac{\gamma'^2}{36}\right\}^3-1\right]^2\left(\frac{1+\rho}{1-\rho}\right)$$

$$\text{where } \gamma'=\gamma\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right) \quad \text{(A7)}$$

$$\text{Let } w=\frac{1}{(\mu-D)\gamma'^2}\left[\left\{1+\frac{\gamma' z}{6}-\frac{\gamma'^2}{36}\right\}^3-1\right]^2\left(\frac{1+\rho}{1-\rho}\right) \quad \text{(A8)}$$

$$C=w\sigma^2 \quad \text{(A9)}$$

$$\frac{\partial C}{\partial\sigma}=2w\sigma\rightarrow\frac{\partial C}{C}=2\frac{\partial\sigma}{\sigma} \quad \text{(A10)\&(A11)}$$

To examine the sensitivity of a storage estimate to the *coefficient of skewness* γ, we consider Eq. (A1) but first let

$$q=\sigma^2\left(\frac{1+\rho}{1-\rho}\right)(\mu-D)\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right)^{-2} \quad \text{(A12)}$$

$$C=q\frac{1}{\gamma^2}\left[\left\{1+\frac{\gamma p}{6}z-\frac{\gamma^2p^2}{36}\right\}^3-1\right]^2$$

$$\text{where } p=\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right) \quad \text{(A13)}$$

$$\frac{\partial C}{\partial\gamma}=q[\,]^2\frac{\partial\frac{1}{\gamma^2}}{\partial\gamma}+q\frac{1}{\gamma^2}\frac{\partial[\,]^2}{\partial\gamma}\rightarrow\frac{\partial C}{\partial\gamma}=-2\frac{q}{\gamma^3}[\,]^2+\frac{q}{\gamma^2}\frac{\partial[\,]^2}{\partial\gamma} \quad \text{(A14)\&(A15)}$$

Consider only the derivative in second term of Eq. (A15)

$$\frac{\partial[\,]^2}{\partial\gamma}=\frac{\partial\left[\left\{1+z\frac{\gamma p}{6}-\frac{\gamma^2p^2}{36}\right\}^3-1\right]^2}{\partial\gamma} \quad \text{(A16)}$$

$$\frac{\partial[\,]^2}{\partial\gamma}=\left(zp-\frac{\gamma p^2}{3}\right)[\,]\{\,\}^2 \quad \text{(A17)}$$

Substitute back into Eq. (A15) gives

$$\frac{\partial C}{\partial\gamma}=-2\frac{q}{\gamma^3}[\,]^2+\frac{q}{\gamma^2}\left(zp-\frac{\gamma p^2}{3}\right)[\,]\{\,\}^2\rightarrow\frac{\partial C}{C}=\frac{-2\frac{q}{\gamma^3}[\,]^2+\frac{q}{\gamma^2}\left(zp-\frac{\gamma p^2}{3}\right)[\,]\{\,\}^2}{q\frac{1}{\gamma^2}[\,]^2}\gamma\frac{\partial\gamma}{\gamma} \quad \text{(A18)\&(A19)}$$

$$\frac{\partial C}{C}=\left(\gamma\left(zp-\frac{\gamma p^2}{3}\right)\left\{1+z\frac{\gamma p}{6}-\frac{\gamma^2p^2}{36}\right\}^2\times\left[\left\{1+z\frac{\gamma p}{6}-\frac{\gamma^2p^2}{36}\right\}^3-1\right]^{-1}-2\right)\frac{\partial\gamma}{\gamma} \quad \text{(A20)}$$

$$\text{where } p=\left(\frac{1-\rho^3}{(1-\rho^2)^{1.5}}\right)$$

To examine the sensitivity of a storage estimate to *autocorrelation* ρ, we consider Eq. (A1)

$$\text{Let } g=\frac{\gamma(1-\rho^3)}{(1-\rho^2)^{1.5}} \quad \text{and} \quad a=\frac{\sigma^2}{(\mu-D)} \quad \text{(A21)\&(A22)}$$

$$C=a\left[\left\{1+\frac{gz}{6}-\frac{g^2}{36}\right\}^3-1\right]^2\left(\frac{1+\rho}{1-\rho}\right)\frac{1}{g^2} \quad \text{(A23)}$$

$$\text{Call} \quad C=a\cdot b(\rho)\cdot c(\rho)\cdot d(\rho) \quad \text{then} \quad \text{(A24)}$$

$$\frac{\partial C}{\partial\rho}=a\frac{\partial b}{\partial\rho}cd+ab\frac{\partial c}{\partial\rho}d+abc\frac{\partial d}{\partial\rho} \quad \text{(A25)}$$

Which, after rearrangement to obtain the relative changes, becomes

$$\frac{\partial C}{C} = \frac{\partial \rho}{\rho} \cdot \rho \left[\left(\frac{\partial b}{\partial g} \frac{1}{b} + \frac{\partial d}{\partial g} \frac{1}{d} \right) \frac{\partial g}{\partial \rho} + \frac{\partial c}{\partial \rho} \frac{1}{c} \right] \tag{A26}$$

Consider b, d and c in turn

$$\text{First term } b(\rho) = \left[\left\{ 1 + \frac{gz}{6} - \frac{g^2}{36} \right\}^3 - 1 \right]^2 \tag{A27}$$

$$\frac{\partial b}{\partial g} = 2 \left[\left\{ 1 + \frac{gz}{6} - \frac{g^2}{36} \right\}^3 - 1 \right] \cdot 3 \cdot \left\{ 1 + \frac{gz}{6} - \frac{g^2}{36} \right\}^2 \left(\frac{z}{6} - \frac{2g}{36} \right) \rightarrow \tag{A28}$$

$$\frac{\partial b}{\partial g} \frac{1}{b} = \left\{ 1 + \frac{gz}{6} - \frac{g^2}{36} \right\}^2 \left(z - \frac{g}{3} \right) \left[\left\{ 1 + \frac{gz}{6} - \frac{g^2}{36} \right\}^3 - 1 \right]^{-1} \tag{A29}$$

$$\text{Second term } d(\rho) = \frac{1}{g^2} \tag{A30}$$

$$\frac{\partial d}{\partial g} = -\frac{2}{g^3} \rightarrow \frac{\partial d}{\partial g} \frac{1}{d} = -\frac{2}{g} \tag{A31)\&(A32}$$

$$\text{Third term } c(\rho) = \frac{1+\rho}{1-\rho} \tag{A33}$$

$$\frac{\partial c}{\partial \rho} = \frac{2}{(1-\rho)^2} \rightarrow \frac{\partial c}{\partial \rho} \frac{1}{c} = \frac{2}{(1-\rho^2)} \tag{A34)\&(A35}$$

$$\text{We also need } \frac{\partial g}{\partial \rho} \quad \text{where } g = \gamma \left(1-\rho^3\right)\left(1-\rho^2\right)^{-1.5} \tag{A36}$$

$$\frac{\partial g}{\partial \rho} = 3\gamma\rho(1-\rho)\left(1-\rho^2\right)^{-2.5} \tag{A37}$$

Substituting Eqs. (A29), (A32), (A35) and (A37) into Eq. (A26) yields

$$\frac{\partial C}{C} = \rho \left[\left(\left\{ 1 + \frac{gz}{6} - \frac{g^2}{36} \right\}^2 \left(z - \frac{g}{3} \right) \left[\left\{ 1 + \frac{gz}{6} - \frac{g^2}{36} \right\}^3 - 1 \right]^{-1} - \frac{2}{g} \right) 3\gamma\rho(1-\rho)\left(1-\rho^2\right)^{-2.5} + \frac{2}{(1-\rho^2)} \right] \frac{\partial \rho}{\rho} \tag{A38}$$

which, if $\rho = 0$, becomes

$$\frac{\partial C}{C} = 2\partial\rho. \tag{A39}$$

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