Introduction to linear programming

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Forest ressource management

- Forest resource management is the art and science of making decisions with regard to the organization, use, and conservation of forests and related resources.
- Forests may be actively managed for timber, water, wildlife, recreation, or a combination thereof.
- Forest resource managers must make decisions affecting both the very long-term future of the forest and day-to-day activities.

Linear programming

- To make decisions, forest managers use models. Models are abstract representations of the real world that are useful for purposes of thinking, forecasting, and decision making.
- Some problems in forest ressource management can be formulated as linear programming models.
- One can recognize three elements in model development: problem definition, model building, and model implementation.

Linear programming

A linear programming problem (LP) is an optimization problem for which we do the following:

- We attempt to maximize (or minimize) a linear function of the decision variables. The function that is to be maximized or minimized is called the objective function.
- The values of the decision variables must satisfy a set of constraints. Each constraint must be a linear equation or linear inequality.
- A sign restriction is associated with each variable. For any variable the sign restriction specifies that it must be either nonnegative or unrestricted in sign.

Linear programming: example 1 - problem definition

The protagonist is a poet-forester who lives in the woods of Northern Wisconsin and 10 years ago bought 90 hectares (ha) of woods. The poet finds that sales from the woods come in very handy to replenish a sometimes-empty wallet. So, he has firmly decided to get the most he can out of his woods. To develop his model the poet has put together the following information:

- 40 ha of the land are covered with red pine plantations, the other 50 ha contain hardwoods;
- Since he bought these woods he has spent approximately 800 days managing the red pine and 1500 days on the hardwoods;
- The poet does not want to spend more than half of his time in the woods;
- The total revenue from his forest during 10 years was \$36,000 from the red pine land and \$60,000 from the northern hardwoods.

Linear programming: example 1 - model building

- Decision variables
- Objective function
- Constraints
  - land constraints
  - working time constraints
  - sign constraints
- Final model

See *Joseph Buongiorno, J. Keith Gilless. Decision Methods for Forest Resource Management, Academic Press, 2003*, for a detailed description of this example.

ASSUMPTIONS OF LINEAR PROGRAMMING

PROPORTIONALITY:

- The contribution of the objective function from each decision variable is proportional to the value of the decision variable. For example, in the poet forester problem, the contribution of red pine management to revenues is proportional to the area of red pine being managed.
- The contribution of each variable to the left-hand side of each constraint is proportional to the value of the variable. For example, the time the poet must put in managing his land is proportional to the area being managed.

ASSUMPTIONS OF LINEAR PROGRAMMING

ADDITIVITY:

- The contribution to the objective function for any variable is independent of the values of the other decision variables.
- The contribution of a variable to the left-hand side of each constraint is independent of the values of the variable.

This means that the contribution of each variable does not depend on the presence or absence of the others. In example 1, regardless of what the poet-forester does with his northern hardwoods, he will always get \$90 per hectare from each hectare of managed red pine, and it will still take him 2 days per hectare per year to manage.

ASSUMPTIONS OF LINEAR PROGRAMMING

DIVISIBILITY:

- Decision variables are continuous.

CERTAINTY:

- A linear programming model is deterministic. It considers that all the parameters are known with certainty

LP: example 2 - Keeping the river clean

A pulp mill makes mechanical and chemical pulp and during the production process it pollutes the river in which it spills its spent waters. The owners would like to minimize pollution, keeping at least 300 people employed at the mill and generating at least 40000€ of revenue per day.

- The maximum capacity of the mill is 300 tons per day to make mechanical pulp and 200 tons per day to make chemical pulp (The two manufacturing processes are independent);
- Both mechanical and chemical pulp require the labor of 1 worker for about 1 day, or 1 workday (wd), per ton produced;
- Pollution is measured by the biological oxygen demand (BOD). 1 ton of mechanical pulp produces 1 unit of BOD, 1 ton of chemical pulp produces 1.5 units;
- The mechanical pulp sells at 100€ per ton, the chemical pulp sells at 200€.

Bibliography

- Wayne L. Winston (Second Edition). *Introduction to Mathematical Programming, Applications and Algorithms*, Duxbury Press.
- Joseph Buongiorno, J. Keith Gilless. *Decision Methods for Forest Resource Management*, Academic Press, 2003.